

# Optimal Location of PMUs for Complete Observability of Power System Sub Network

Gomathi Venugopal<sup>1</sup>, Ramachandran Veilumuthu<sup>2</sup>, and Chellammal Arumugam<sup>3</sup>

<sup>1,3</sup> College of Engineering, Anna University/Department of Electrical and Electronics Engineering, Chennai, India  
Email: gomesvg@annauniv.edu, chell\_meena@yahoo.co.in

<sup>2</sup> College of Engineering, Anna University/Department of Computer Science and Engineering, Chennai, India  
Email: rama@annauniv.edu

**Abstract**— Improvements in power system control and protection is achieved by utilizing real time synchronized phasor measurements. The trend in recent years is the steady increase of Phasor Measurement Unit (PMU) installations worldwide for various applications those targeted for State Estimation enhancement. This paper solves the optimal placement of phasor measurement units for complete observability using Integer Linear Programming (ILP) solution methodology. ILP is used to determine the minimum optimal number and location of PMUs needed to make the network observable. The method is tested on the Tamil Nadu Electricity Board (TNEB) 110 KV (North and South), 230 KV sub network of Chennai and 400 KV network of Southern Region and complete system observability has been validated.

**Index Terms**—Integer Linear Programming (ILP), PMU placement, State Estimation, Complete Observability

## I. INTRODUCTION

State estimation is the main composition of the modern energy management system (EMS). It gives real time status for power system analysis and control. The traditional State Estimation (SE) utilizes only the measurements from the SCADA (Supervisory Control and Data Acquisition system). The PMU measurements have now become another important data source for SE. Improvements in power system control and protection is achieved by utilizing real time synchronized phasor angle measurements. The location for Phasor Measurement Unit is to be found out preserving the system Complete Observability of the Network. In this paper an attempt is made to suggest the optimal placement of PMU for the sub network of Tamil Nadu and Southern Region. PMUs are considered as a promising tool for future monitoring, protection and control of power systems. The PMU placement problem is formulated as a binary integer linear programming, in which the binary decision variables (0, 1) determine whether to install a PMU at each bus.

In [1], a phasor measurement placement method based on the topological observability theory using graph theorem analysis is proposed. A minimal number of buses with measurements is found through both a modified bisecting search and simulated annealing-based method. However, the possible contingency in the power system is not considered, the measurement

set is not robust to loss of measurements and branch outages. In [2], an optimal PMU placement method based on the nondominated sorting genetic algorithm (GA) is proposed.

The problem is to find the placement of minimum PMUs, so that the system is observable during its normal operation and any single-branch contingency. Each optimal solution of objective functions is estimated by the graph theory and simple GA. Then, the best tradeoff between competing objectives is searched by using nondominated sorting GA. Since this method requires more complexity computation, it is limited by the size of the problem.

In [3], the integer programming based on network observability and the cost of PMUs has been applied to find the PMU placement. This method can be applied to the case of the mixed measurement set in which PMUs and conventional measurements are employed in the system. These papers find the minimal buses where PMUs should be installed, such that the power network is observable. It is assumed that the installed PMUs have enough channels to record the bus voltage phasor at their associated buses and current phasors along all branches that are incident to the buses. However, the topological observability does not guarantee that the state estimation can be solved [4]. As a result, the computed solution may not be accurate due to round off error during numerical computation. The optimal placement methods for conventional measurements against contingency have been addressed in [5]–[7].

A sequential selection process based on measurement sensitivities and measurement failures performance indices has been presented in [5]. A topological method considering only single-branch outage is presented in [6]. In addition, a numerical algorithm to optimally upgrade the existing measurements in order to make the network observable under loss of single measurement and any single-branch outage is proposed in [7]. In [11] the authors use integer programming to determine the minimum number of PMUs.

In this paper, an Integer Linear programming is used to determine the optimal number and location of PMUs to make the system measurement model observable and thereby it can be used for power system state estimation. This method is applied for the Sub network

of Tamil Nadu Electricity Board (TNEB) System namely the 110KV (North and South), 230KV Chennai Network and 400KV Southern Region network.

## II. PHASOR MEASUREMENT UNIT

Phasor Measurement Unit (PMU) – a device which by employing widely used satellite technology offers new opportunities in power system monitoring, protection, analysis and control. PMUs facilitate innovative solutions to traditional utility problems and offer power system engineers a whole range of potential benefits, including precise estimates of the power system state can be obtained at frequent intervals, enabling dynamic phenomena to be observed from a central location and appropriate control actions taken. Post-disturbance analyses are much improved because precise snapshots of the system states are obtained through GPS synchronization. Advanced protection based upon synchronized phasor measurements could be implemented; with options for improving overall system response to catastrophic events. Advanced control using remote feedback becomes possible, thereby improving controller performance.

PMU measures voltage and current phasor in a power system. Synchronism among phasor measurements is achieved by same-time sampling of voltage and current waveforms using a common synchronizing signal from the global positioning satellite (GPS) [8]–[10]. PMUs have higher accuracy than conventional measurements. They reduce effects of time-skew among measurements, useful for many other applications such as system protection, control and stability assessment, aid topology error identification, parameter error detection and correction and improve accuracy of state estimation.

The introduction of PMUs in power systems significantly improves the possibility for monitoring and analyzing power system dynamics. A number of synchronized phasor measurement terminal installed in different locations of a power system provides important information about different AC quantities e.g. voltages, currents, active and reactive power, all of them based on the same GPS time reference. Synchronized measurements make it possible to directly measure phase angles between corresponding phasors in different locations within the power system. Improved monitoring and remedial action capabilities allow network operators to utilize the existing power system in a more efficient way. Improved information allows fast and reliable emergency actions, which reduces the need for relatively high transmission margins required by potential power system disturbances. Instead of merely surviving the worst credible contingency the power system should survive the worst credible contingency followed by remedial actions initiated by various new functions based on

phasor measurement. Thus Phasor measurement opens a wide range of new applications, like monitoring and recording of power system dynamics, improved SE, system wide power oscillation mitigation, robust two side transmission line fault locator, emergency control during large disturbances, voltage control in a power system and synchronised event recording.

## III. PROBLEM FORMULATION

The Optimal placement of PMU becomes an important problem to be solved in power system state estimation. The PMU placement problem is formulated as a binary integer linear programming, in which the binary decision variables (0, 1) determine whether to install a PMU at each bus, while preserving the system observability and lowest system metering economy. It is neither economical nor necessary to install a PMU at every node of a wide-area interconnected network.

The cost of a PMU depends on a number of factors, including the number of measuring terminals (channels), CT and PT connections, power connection, station ground connection, and GPS antenna connection. The main purpose of performing PMU placement problem is to minimize the number of installed PMUs, so for n-bus system the optimization problem is given as:

$$\begin{aligned} &\text{Minimize } \sum_i^n w_i x_i \\ &\text{Subject to } f(X) \geq \hat{1} \end{aligned}$$

Where  $w_i$  is the installation cost of the PMU at bus

i. Assume  $w_i = 1$

$$x_i = 1, \text{ if a PMU installed at bus } i \\ 0, \text{ Otherwise}$$

$f(X)$  is a vector function representing the constraints

$\hat{1}$  is a vector whose entries are all equal to 1.

## IV Case Study And Results

The Spanning Tree for different Sub networks of Tamil Nadu (110 KV, 230 KV) and 400 KV Southern Region are drawn and the Multi partitioning algorithm is applied to form various blocks. For each block the objective function and the constraint equations are formulated and solved using ILP.

The 110KV (North) sub network of Tamil Nadu is shown in Figure.1. The Spanning Tree for the network is shown in Figure 4. Multi partitioning algorithm is applied and thus various blocks are formed. Similarly Figure 2, 3 and 7 represent the 110KV (South), 230 KV and 400 KV network of Southern Region. The Spanning tree is shown in Figure 5, 6 and 8



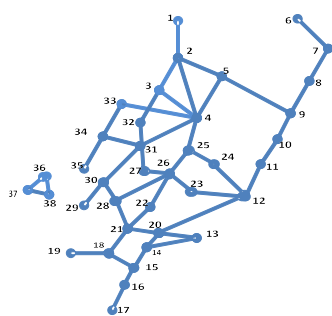


Figure 8. Spanning Tree for 400KV Network of Southern Region

TABLE I  
Optimal number and Location of PMUs

Network	Number of PMUs	Location of PMUs
110 KV (North)	16	6-8-9-11-13-15-20-24-27-29-32-40-42-45-47-48
110 KV (South)	14	2-7-14-15-16-21-27-28-32-36-43-44-48-50
230 KV	3	3-6-8
400 KV	10	2-6-9-17-18-20-26-29-34-36

## V. OBSERVABILITY ANALYSIS

Observability analysis is a fundamental component of real-time state estimation. Observability analysis of electric power system is to study whether there is enough measurement in system in order to estimate the state of electric power system including the amplitude value and phase angle of the voltage.

Observability analysis mainly contains two methods: numerical method and topological method. The topological methods are based on whether a spanning tree of full rank can be constructed. The numerical methods rely on whether the measurement information gain or Jacobian matrix is of full rank. If the voltage of a node can be measured directly or can be calculated by other voltage phasor and current phasor, the node is observable. The Topological Observability uses ILP concepts to find the locations for the PMU placement (as shown in Table I) and thus, to make the system completely observable. The rules for PMU based topological observability is given as follows.

## A. Rules for PMU Based Topological Observability Analysis

For making the system topologically observable using PMUs, following simple rules are used,

1. If voltage phasor and current phasor at one end of a branch are known, voltage phasor at the other end of the branch can be calculated using Ohm's law.
2. If voltage phasors at both the ends of a branch are known, branch current can be calculated.
3. If there is a zero injection bus without a PMU, whose outgoing currents are known except for one, then the unknown outgoing current can be calculated using Kirchhoff's Current Law (KCL).

## B. Validation of Observability for different power system sub networks:

Complete observability refers to the PMU placement scenario when the number and location of the PMUs are sufficient to determine the complete set of bus voltages of the state of the network considered (230 KV network). Fig.9. shows the Directly Observable and Indirectly Observable nodes for 230KV Network of Chennai.

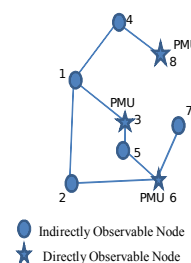


Figure 9. Directly Observable and Indirectly Observable Nodes for 230KV Network of Chennai.

The voltage at buses 3, 6 and 8 are directly measured (Directly Observed) using PMUs, while voltages at buses 1, 2, 4, 5 and 7 can be calculated using the measured voltages and line currents.

In directly Observable nodes, voltages and line currents are directly measured. In an indirectly observable node or calculated bus, the voltages and line currents are calculated from the PMU measurements of the buses linked to them. Thus the network becomes completely observable.

Similarly, the above Observability Check is done for 110KV (North, South) Network of Chennai and 400KV Network of Southern Region and found to be fully observable with a total number of 16, 14 and 10 optimally placed PMUs respectively. (Refer TABLE I)

## CONCLUSION

The accuracy of PMU measurement is high. There is a substantial contribution of PMU to power system state estimation. In practical, power system measurement from SCADA to be sent to the control centre requires 2s whereas the measurement from PMU needs 40 ms to reach the control centre. There will 50

times measurement from PMU sent to the control centre in the interval that two measurements from SCADA will be received by the centre [13]. Thus the remedial actions in case of a contingency can be carried out more faster and effectively. It is no doubt that direct measurement of state can improve the observability and the accuracy of state estimation for the Real-time network. This paper solves a generalized integer linear programming formulation for optimal PMU placement for the sub network of Tamil Nadu 110 KV (North and South), 230 KV sub network of Chennai and 400 KV network of Southern Region. An Algorithm to combine all the networks of the TNEB systems to obtain the global optimal number and location of PMUs is to be proposed. To arrive at a more robust solution, we need to consider the possible contingencies in the power system network considered. Integrating the PMU data, various EMS security functions can be carried out effectively.

#### ACKNOWLEDGMENT

The authors acknowledge the support rendered by the Tamil Nadu Electricity Board towards providing with the facts and figures.

#### REFERENCES

- [1] T. L. Baldwin, L. Mili, M. B. Boisen, Jr, and R. Adapa, "Power system observability with minimal phasor measurement placement," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 707–715, May 1993.
- [2] B. Milosevic and M. Begovic, "Nondominated sorting genetic algorithm for optimal phasor measurement placement," *IEEE Trans. PowerSyst.*, vol. 18, no. 1, pp. 69–75, Feb. 2003.
- [3] Y. M. Park, Y. H. Moon, J. B. Choo, and T. W. Kwon, "Design of reliable measurement system for state estimation," *IEEE Trans. PowerSyst.*, vol. 3, no. 3, pp. 830–836, Aug. 1988.
- [4] A. Abur and F. H. Magnago, "Optimal meter placement for maintaining observability during single branch outage," *IEEE Trans. Power Syst.*, vol. 14, no. 4, pp. 1273–1278, Nov. 1999.
- [5] F. H. Magnago and A. Abur, "A unified approach to robust meter placement against loss of measurements and branch outages," *IEEE Trans. Power Syst.*, vol. 15, no. 3, pp. 945–949, Aug. 2000.

- [6] R. F. Nuqui and A. G. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability," *IEEE Trans. Power Del.*, vol. 20, no. 4, pp. 2381–2388, Oct. 2005.
- [7] A. G. Phadke, "Synchronized phasor measurements in power systems," *IEEE Compute. Appl. Power*, vol. 6, no. 2, pp. 10–15, Apr. 1993.
- [8] A. G. Phadke, J. S. Thorp, and K. J. Karimi, "State estimation with phasor measurements," *IEEE Trans. Power Syst.*, vol. 1, no. 1, pp.3074-3079, April 2004
- [9] B.W Kernighan and L.Lin "An efficient heuristic procedure for partitioning graphs", *Bell Syst. Tech. Journal.*, vol.49, Feb1979, pp.291-307.
- [10] A. Monticelli, "State Estimation in Electric Power Systems: A Generalized Approach". Norwell,MA: Kluwer, vol 2,no.2,Nov 1999.
- [11] Hui Xue, Qing-quan Jia, Ning Wang, Zhi-qianBo, Hai-tang Wang, Hong-xia Ma, "A Dynamic State Estimation with PMU and SCADA measurement for Power systems,"8<sup>th</sup> International Power Engineering Conference (IPEC 2007) vol 7, no.5, pp 435-440, Oct2003
- [12] B. Xu and A. Abur, "Optimal placement of phasor measurement units for state estimation," *Final Project Report, PSERC*, vol1, pp 534-540, Oct. 2005.
- [13] B. Xu and A. Abur, "Observability analysis and measurement placement for system with PMUs," in *Proc. IEEE Power System Conf. Expo.*, Oct. 2004, vol. 2, pp. 943–946.

#### BIOGRAPHIES

Gomathi Venugopal received the Bachelors degree from University of Madras, in 2002. Received the Masters degree from College of Engineering, Anna University Chennai in 2004. She is presently working as a Lecturer in College of Engineering, Anna University, Chennai. Her fields of interest include Power System Control and Operation, Service Oriented Architecture and Web Services.

Ramachandran Veilumuthu received his Masters degree and Ph.D in Electrical Engineering from College of Engineering, Anna University, Chennai, India. He is currently working as a Professor in the Department of Computer Science and Engineering, College of Engineering, Anna University, Chennai. His research interests include power system reliability engineering, network security, soft computing and Web technology.

Chellammal Arumugam received her Bachelors degree from Vel Tech Engineering College, Avadi in 2006. She is pursuing her Masters in Power Systems Engineering, College of Engineering, Anna University, Chennai.